XIV. *A Letter from the Rev. Mr.* Richard Dunthorne *to the Reverend Mr.* Richard Maſon *F. R. S. and Keeper of the* Woodwardian Muſeum *at* Cambridge, *concerning the* Acceleration *of the* Moon.

SIR, *Cambridge, Feb.* 28, 1748-9.

Read June 1. 1749. AFTER I had compared a good Number of modern Obſervations made in different Situations of the Moon and of her Orbit in reſpect of the Sun, with the *Newtonian* Theory, as in my Letter of *Nov.* 4, 1746;† I proceeded to examine the mean Motion of the Moon, of her Apogee, and Nodes, to ſee whether they were well repreſented by the Tables for any conſiderable Number of Years, and whether I ſhould be able to make out that Acceleration of the Moon's Motion which Dr. *Halley* ſuſpected. *Vide* Phil. Tranſ. *n.* 218.

To this End I compared ſeveral Eclipſes of the Moon obſerved by *Tycho Brahe*, as they are ſet down in his *Progymnaſmata*, *p.* 114, with the Tables *, and found them agree full as well as could be expected; conſidering the Imperfection of his Clocks, and the Difficulty there muſt commonly have been in determining the Middle of the Eclipſe from the Facts obſerved, as publiſhed in his *Hiſtoria Cœleſtis*. Indeed the ſmall Diſtance of Time between *Tycho Brahe* and

† See theſe Tranſact. No. 482. p. 412.

* My Tables corrected as in my former Letter; which is always to be underſtood of the Tables mention'd in this.

and *Flamſteea* render'd *Tycho*'s Obſervations but of little Uſe in this Enquiry.

The next Obſervations that occurred to me were thoſe of *Bernard Walther* and *Regiomontanus*, which being at double the Diſtance of Time from *Flamſteed* that *Tycho*'s were, ſeemed to promiſe ſome Aſſiſtance in this Matter: Upon comparing ſuch of their Eclipſes of the Moon whoſe Circumſtances are beſt related with the Tables, I found the computed Places of the Moon were moſtly 5′ too forward, and in ſome conſiderably more, which I could hardly perſuade myſelf to throw upon the Errors of Obſervation; but concluded, that the Moon's mean Motion ſince that time, muſt have been ſomething ſwifter than the Tables repreſent it; though the Diſagreement of the Obſervations between themſelves is too great to infer any thing from them with Certainty in ſo nice an Affair.

Then I compared the four well-known Eclipſes obſerved by *Albategnius* with the Tables, and found the computed Places of the Moon in three of them conſiderably too forward: This, if I could have depended upon the Longitude of *Aracta*, would very much have confirmed me in the Opinion, that the Moon's mean Motion muſt have been ſwifter in ſome of the laſt Centuries than the Tables make it; though the Differences between theſe Obſervations, and the Tables, are not uniform enough to be taken for a certain Proof thereof.

I could meet with no Obſervations of Eclipſes to be at all depended upon between thoſe of *Regiomontanus* and *Albategnius*, except two of the Sun and one of the Moon made at *Cairo* in *Egypt*,

related in the *Prolegomena* to *Tycho Brahe's Hiſtoria Cœleſtis*, *p.* 34; nor any between thoſe of *Albategnius* and *Ptolemy*, beſides the Eclipſe of the Sun obſerved by *Theon* at *Alexandria*; notwithſtanding I carefully ſearched all the Remains of Antiquity I could find with that View. Theſe Eclipſes of the Sun are the more valuable, becauſe they were obſerved in Places the Longitudes and Latitudes whereof are determined by Monſieur *Chazelles* of the *Royal Academy* of *Sciences*, who was ſent by the *French* King in the Year 1693, with proper Inſtruments for that Purpoſe. *Du Hamel Hiſt. Acad. p.* 309, 395.

The ſolar Eclipſe obſerved by *Theon* was in the 112th Year of *Nabonaſſar* the Day of *Thoth*, according to the *Egyptians*, but the 22d Day of *Pauni*, according to the *Alexandrians*: He carefully obſerved the Beginning of 2 temporal Hours and 50′ Afternoon, and the End at 4½ Hours nearly Afternoon at *Alexandria*. *Theonis Comment. in Ptol. mag. Conſtruct. p.* 332. This Eclipſe was *June* 16, in the Year of Chriſt 364: And the temporal Hour at *Alexandria* being at that time to the equinoctial Hour as 7 to 6, makes the Beginning at 3 equinoctial Hours and 18′ Afternoon, and the End at 5 equinoctial Hours 15′ nearly.

The Eclipſes obſerved at *Grand Cairo* were as follows.

" Anno Hegiræ 367, die *Jovis*, qui erat 28, rabie
" poſterioris (is eſt ordine menſis quartus, et incipit
" ille annus *Saracenicus* die 19 *Auguſti*, anno *Chri-*
" *ſtiano* 977) obſervatum fuit *Cahiræ* in *Ægypti*
" metropoli initium eclipſis ſolaris, cum altitudo ſolis
" eſſet

" eſſet 15° 43′. quantitas obſcurationis 8 digit. Ea " finita, ſol elevabatur $33\frac{1}{2}$ *gr. Ex Schickardo in* " *MS.*" This Eclipſe was *Decemb.* 13, in the Year of Chriſt 977, the Beginning at 8^h 25′, and the End at 10^h 45′ apparent Time in the Morning.

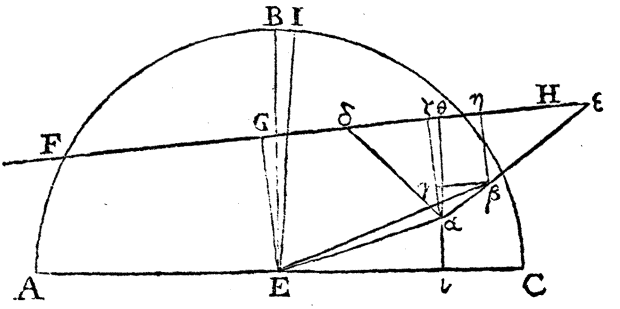
" Anno eodem die Sabbathi, videlicet 29 menſis " *Sywal* (numero decimi, qui Paſchalis eſt eorum) " eclipſis Solis occupavit digitos $7\frac{1}{2}$. In principio " Sol altus fere 56°. In fine Sol occiduus elevaba- " tur gradibus 26. *Ex Schickardo in MS.*" —— This Eclipſe was *June* 8, in the Year of Chriſt 978. The Beginning at 2^h 31′, and the End at 4^h 50′ apparent Time Afternoon.

" Anno Hegiræ 368 (qui incœpit die 9 *Auguſti*, " anno *Chriſtiano* 978) die *Jovis*, 14 *Sywal*, Luna " fuit orta cum defectu, qui ad $5\frac{1}{2}$ digitos accrevit; " cum extaret ſupra horizontem gradibus etiam 26 " ſubaudio finem tunc accidiſſe). *Schickardus.*"---- This Eclipſe was *May* 14, in the Year of Chriſt 979; but as the Middle cannot be known from what was obſerved of it, I made no uſe thereof in this Enquiry. The Account concludes with the following Paragraph:

" Hæ tres obſervationes habitæ ſunt ab *Ibn-Junis*, " qui juſſu Regis *Abu-Haly Almanzor*, ſapientis, " *Ægypto* tunc Imperantis, rebus vacabat cœleſ- " tibus. Hujus authoris tabulas habet *Jac. Golius* " Profeſſor *Lugdun.* (qui mihi inde communicavit " iſta) in quibus plures aliæ, ſui et ſuperioris ævi ob- " ſervationes extant. Locus obſervationis propinquus " urbi *Cahiro. Schickardus.*"

That

That the before-mention'd ſolar Eclipſes might be applied to the Examination of the Lunar Motions, I contrived the following Method; which I think renders Eclipſes of the Sun as uſeful at leaſt as thoſe of the Moon are in that Buſineſs.



Let *ABC* in the annexed Figure repreſent half the Earth's enlightened Disk, *AEC* a Portion of the Ecliptic projected thereon *FGH* the Path of the Moon's Shadow over the Disk, *EI*, the univerſal Meridian, α the Situation of the Place at the Beginning of the Eclipſe, β its Situation at the End thereof, δ the Centre of the Shade at the Beginning, and ε its Centre at the End of the Eclipſe. Draw *EG*, $\alpha\zeta$, and $\beta\eta$, perpendicular to the Path of the Shadow, $\beta\gamma$ parallel thereto; join $\alpha\delta$ and $\beta\varepsilon$, and through α draw $\theta\alpha\iota$ perpendicular to *AC*.

Then (computing the true Places of the Sun and Moon at the obſerved Times of the Beginning and End of the Eclipſe) we ſhall have given $\delta\varepsilon$ the Motion of the Moon from the Sun in her Orbit during the Time of the Eclipſe, and $\alpha\delta = \beta\varepsilon$ the Semidiameter of the *Penumbra*; which are to be reduced into ſuch Parts as the Semidiameter of the Disk contains

tains 10000: The Angles *BEI* and *BEG*, being found by Methods commonly known, *GEI* their Sum or Difference will be likewiſe given. Alſo $E\alpha$ and $E\beta$ will be Sines of the Sun's Altitude at the Beginning and End of the Eclipſe reſpectively; $IE\alpha$ and $IE\beta$ are the Angles at the Sun between the Vertex of the Place and the Pole of thoſe Times; which being found, the Angle $\alpha E\beta$, their Difference will be known, from whence the Line $\alpha\beta$ and the Angle $E\alpha\beta$ may be computed.

The Angle $GE\alpha$ is the Sum or Difference of the known Angles *GEI* and $IE\alpha$: In the Figure before us, the Complement of this to a Semicircle is $E\alpha\gamma$; which being ſubtracted from $E\alpha\beta$ leaves the Angle $\gamma\alpha\beta$, from whence and the Line $\alpha\beta$, $\alpha\gamma$, and $\gamma\beta = \zeta\eta$ may be found.

Let $a = \delta\varepsilon - \zeta\eta$, $b = \alpha\delta = \beta\varepsilon$, $c = \alpha\gamma$, and $x = \beta\eta = \gamma\zeta$.

Then $\sqrt{bb - xx} = \eta\varepsilon$, and $\sqrt{bb - cc - 2cx - xx} = \delta\zeta$, by *Eucl.* 1.47.

Conſequently $a - \sqrt{bb - xx} = \sqrt{bb - cc - 2cx - xx}$ which being reduced, gives us the quadratic Equation $xx + cx = \frac{4a^2b^2 - a^4 - 2a^2c^2}{4aa + 4cc}$. This Equation ſolved, gives us the Value of x, from which $\delta\zeta$ and $\eta\varepsilon$ will be likewiſe had. In the Triangle $\alpha\zeta\theta$ we have $\alpha\zeta$ and the Angle $\zeta\alpha\theta = GEB$ given, whence $\alpha\theta$ and $\zeta\theta$ may be found: Conſequently $\delta\theta$ will be known; and from the obſerved Time of the Beginning of the Eclipſe, and hourly Motion of the Moon from the Sun, the Time when the Centre of the Shade is at θ will be had. Laſtly, in the Triangle $E\iota\alpha$, we have given the Side $E\alpha$, and the Angle $E\alpha\iota = BE\alpha$

BEα (the Sum or Difference of the Angles BEI and IEα); therefore the Sides Eι and $\alpha\iota$ may be found. But Eι is the Diſtance of the Moon from the Sun in the Ecliptic, and $\alpha\iota \pm \alpha\theta$ the Moon's Latitude at the Time when the Centre of the Shade is at θ; which may be compared with the Computation from the Tables for that Time.

By this Means I compared the aforeſaid Solar Eclipſes with the Tables, and found the Difference in Longitude and Latitude, as follows.

A.D.	Apparent Time at Greenwich.	Diſt. ☽ a ☉ from Eι.	Lat. ☽ from 0ι.	☽ a ☉ by Tab.	Lat. ☽ by Tab.	Diff. from Obſer. in Long	Diff. from Obſer. in Lat.	Diff. in Lat. from Digits obſerved.
	h. ′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″
364	June 16. 2 4 20	39 41 in conſeq.	34 37 Nor	35 25	37 26 Nor.	−4 16	+2 49	
977	Dec. 12. 19 12 30	43 39 in antec.	30 23 Nor.	36 3	31 50 Nor.	+7 36	+1 27	− 2 36
978	June 8. 1 16 10	29 3 in conſeq.	8 24 Sou.	37 48	3 21 Sou.	+8 45	−5 3	+ 3 38

The Agreement there is between the two laſt of theſe Differences in Longitude, ſhews that the Tables repreſent the mean Motion of the Moon's Apogee very well for above 700 Years, the Moon being very near her Perigee at the Time of one of thoſe Eclipſes, and near her Apogee at the Time of the other.

By the ſame Method I alſo compared the Sun's Eclipſe, *July* 29, 1478. (which appears, from what is related of it, to have been carefully obſerved by *Bernard Walther* at *Nuremberg*), with the Tables, and found the Difference in Longitude to be + 10′ 29″, and in Latitude + 9′ 12″. This wide Difference in Latitude, from the Tables, that agree ſo well with the former ancient Obſervations, confirmed me in the Opinion, that the *Nuremberg* Obſer-

Obſervations are too inaccurate to determine any thing from them in this Affair.

The Eclipſes recorded by *Ptolemy* in his *Almageſt*, are moſt of them ſo looſely deſcribed, that, if they ſhew us the Moon's mean Motion has been accelerated in the long Interval of Time ſince they happened, they are wholly incapable of ſhewing us, how much that Acceleration has been. There are indeed two or three of them attended with ſuch lucky Circumſtances as not only plainly prove, that there has been ſuch an Acceleration, but alſo help us to gueſs at its Quantity. One of theſe is the Eclipſe, ſaid by *Hipparchus* to have been obſerved at *Babylon*, in the 366th Year of *Nabonaſſar*, the Night between the 26th and 27th Days of *Thoth*, when a ſmall Part of the Moon's Diſk was eclipſed from the North Eaſt, half an Hour before the End of the Night, and the Moon ſet eclipſed. This was in the Year before Chriſt 313, *Decemb.* 22. The Middle of this Eclipſe at *Babylon* (ſuppoſing with *Ptolemy* the Meridian of that Place to be 50′ in Time Eaſt of the Meridian of *Alexandria*), by my Tables was *Dec.* 22. 4^h 4′ apparent Time; the Duration was 1^h 37′, *Ptolemy* makes it 1^h 30′ nearly; whence the Beginning ſhould have been about 3^h 15′ after Midnight: According to *Ptolemy*, the Night at *Babylon* was at that Time 14^h 24′ long, and therefore Sun riſe at 7^h 12′ after Midnight; and as the Moon had then South Latitude, and was not quite come to the Sun's Oppoſition, her apparent Setting muſt have been ſomething ſooner, *i. e.* more than an Hour before the Beginning of the Eclipſe, according to the Tables; whereas the Moon was ſeen

eclipſed ſome Time before her Setting; which, I think, demonſtrates, that the Moon's Place muſt have been forwarder, and conſequently her Motion ſince that Time leſs than the Tables make it by about 40′ or 50′. But the computed Place of the Moon in each of the before-mentioned Solar Eclipſes obſerved at *Grand Cairo*, being about 8′ before her Place, from Obſervation ſhews us, that the mean Motion of this Luminary has been ſomething greater in the laſt 700 Years than the Tables ſuppoſe it, and therefore muſt have been accelerated.

This Acceleration is further confirmed by the Eclipſe, which *Hipparchus* ſays was obſerved at *Alexandria*, in the 54th Year of the ſecond *Calippic* Period, the 16th Day of *Meſſori*, when (he ſays) the Moon began to be eclipſed half an Hour before her Riſing, and was wholly clear again in the Middle of the third Hour of the Night. This was in the Year before Chriſt 201. *Sept.* 22. The Middle of this Eclipſe at *Alexandria* by the Tables was *Sept.* 22. 7ʰ 44′ apparent Time; and the Duration 3ʰ 4′, which makes the Beginning at 6ʰ 12′ apparent Time, that is, about 10′ after the riſing of the Moon at *Alexandria*, or 40′ later than the Beginning from Obſervation. This Difference in Time makes a Difference of near 20′ in the Moon's Place.

The moſt antient Eclipſe of which we have any Account remaining, namely that related by *Ptolemy*, to have been obſerved at *Babylon* the firſt Year of *Mardokempad*, in the Night between the 29th and 30th Days of *Thoth*, in which the Moon began to be eclipſed when one Hour after her Riſing was fully paſt; if, by reaſon of the Latitude of the Expreſſion,

preſſion, it be not a direct Proof of the Acceleration, it may nevertheleſs help to limit its Quantity. This Eclipſe was in the Year before Chriſt 721. *March* 19. The Middle whereof at *Babylon*, by the Tables, was *March* 19. 10^h 26′ apparent Time; and the Beginning at 8^h 32′, the apparent Riſing of the Moon at that Place was about 5^h 46′ Afternoon; ſo that the obſerved Beginning of the Eclipſe was at leaſt 6^h 46′ Afternoon, *i. e.* not above $1\frac{1}{4}^h$ before the Beginning, by the Tables: Wherefore the Moon's true Place could precede her Place by Computation but little more than 50′ at that Time.

If we take this Acceleration to be uniform, as the Obſervations whereupon it is grounded are not ſufficient to prove the Contrary, the Aggregate of it will be as the Square of the Time: And if we ſuppoſe it to be 10″ in 100 Years, and that the Tables truly repreſent the Moon's Place about *A. D.* 700. it will beſt agree with the before-mentioned Obſervations; and the Difference between the Moon's Place by the Tables and her Place in the Heavens, will be as follows.

Years before Chriſt.	Error of Tab.	Years of Chriſt.	Error of Tab.	Years of Chriſt.	Error of Tab.
	′ ″		′ ″		′ ″
700	—56 6	200	—12 30	1100	+ 4 0
600	—49 50	300	— 9 20	1200	+ 4 10
500	—44 0	400	— 6 30	1300	+ 4 0
400	—38 30	500	— 4 0	1400	+ 3 30
300	—33 20	600	— 1 50	1500	+ 2 40
200	—28 30	700	0 0	1600	+ 1 30
100	—24 0	800	+ 1 30	1700	0 0
A.D.0	—19 50	900	+ 2 40		
100	—16 0	1000	+ 3 30		

I am,

SIR,

Your humble Servant,

Richard Dunthorne.

XV. Alberti Halleri, *Archiatri Reg. Medicin. Prof.* Gotting. & R. S. Lond. *S.* Fabricæ morboſæ in cadaveribus repertæ hiſtoriæ aliquæ.

OBS. I.

Read, June 8. 1749. IN femina quadragenaria reperi Venam cavam inter renalis ſiniſtræ originem, et inter iliacas venas, enormiter anguſtatam, ut vix quid-